Arbitrarily short coherence length in wave fields within finite lossless source regions

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We show that it is possible to construct stochastic source distributions that generate wave fields with any desired spatial coherence properties inside a prescribed finite region, regardless of the refractive-index distribution therein. In particular, we demonstrate that there is no universal sinc-function form for the field correlations produced by statistically homogeneous and isotropic source distributions within large lossless regions, contrary to what is suggested by previous work. The results also disprove the commonly held belief that in lossless source regions the coherence length of the light is bounded below by the blackbody coherence length—that is, by approximately half the wavelength of the field.

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The spatial coherence properties of random, stationary fields within source regions have recently attracted considerable interest [1–7]. Initially fields produced by δ-correlated sources in lossless spherical regions were examined [1]. As the results suggested that large source regions are well approximated by all space, subsequent studies have concentrated on infinite source domains. In particular, it has been shown that when a source distribution filling all space is statistically homogeneous and isotropic, the coherence function of the generated field is of a universal sinc-function form at the limit of no absorption [2]. That result has been extended to dimensions other than 3 [5] and to vectorial electromagnetic fields [6], but it has also been shown not to generally hold in the presence of losses, no matter how small the absorption is [7]. More precisely, regardless of the weakness of the absorption, it is always possible to find sources of infinite extent that produce wave fields whose coherence length is as short as desired.

In the present paper we show that even for finite-sized lossless regions it is, in fact, straightforward to construct statistically homogeneous and isotropic sources that give rise to fields whose coherence functions do not exhibit the universal form. More generally, an analogous construction can be used to obtain a source distribution (not necessarily isotropic nor homogeneous) that produces a field with any specified spatial coherence properties within a finite-sized region, irrespective of the refractive-index distribution therein. In particular, such a source distribution can generate a wave field with an arbitrarily short coherence length inside a lossless source region. Specifically the coherence length is independent of the wavelength λ at which the field is considered. Therefore, the belief that δ-correlated sources or blackbody radiators must produce the most incoherent fields attainable, suggesting that $\lambda/2$ is a *de facto* lower limit for the coherence length of the field within lossless regions (see, e.g., Refs. [1,3]), is erroneous. It is important to note that the possibility of an arbitrarily short coherence length, as will be reported here, is not an effect that is due to (extreme) losses within the source region, which are known to yield short coherence lengths [7,8]. Instead, it stems from properly tuned interferences between the fields produced in different parts of the source and is therefore an effect that exists for any particular refractive-index distribution.

To avoid unnecessarily complex expressions and thus to bring out the important aspects of the results in a clear and concise manner, we limit our considerations here to scalar electromagnetic fields, but as we note later on, this should not affect the generality of the results. In a scalar treatment each monochromatic component of an electromagnetic field is modeled by a scalar function [9]. Such a scalar field $u(\mathbf{r})$ satisfies an inhomogeneous Helmholtz equation of the form

$$\nabla^2 u(\mathbf{r}) + \kappa^2(\mathbf{r})u(\mathbf{r}) = -4\pi\rho_u(\mathbf{r}), \tag{1}$$

where $\kappa(\mathbf{r})=k_0 n(\mathbf{r})$ with $k_0=2\pi/\lambda_0$ being the vacuum wave number and $n(\mathbf{r})$ the refractive index. Here λ_0 is the vacuum wavelength of the monochromatic field. The function $\rho_u(\mathbf{r})$ in Eq. (1) is the primary source distribution that generates the field $u(\mathbf{r})$. We assume that both this function and $\kappa(\mathbf{r})$ are piecewise continuous. In addition, we consider only finite source and scatterer regions, whereby it follows that there is a finite region $\Omega \subset \mathbb{R}^3$, such that $\rho_u(\mathbf{r})=0$ and $\kappa(\mathbf{r})=k_0$ when $\mathbf{r}\notin\Omega$. Since the field $u(\mathbf{r})$ is generated within the finite region Ω, it is reasonable to require that $u(\mathbf{r})$ carry energy only out of that region—i.e., that the field be *outgoing*. The outgoing solution to Eq. (1) is characterized by the Sommerfeld radiation condition [10,11]

$$\lim_{r\to\infty} r[\partial_r u(\mathbf{r}) - ik_0 u(\mathbf{r})] = 0, \quad \text{unif. } \hat{\mathbf{r}}, \tag{2}$$

where $r=|\mathbf{r}|$ and $\hat{\mathbf{r}}=\mathbf{r}/r$. This condition is sufficient to make the solution $u(\mathbf{r})$ to Eq. (1) unique if $\mathrm{Im}\{\kappa^2(\mathbf{r})\}>0$ wherever $\kappa(\mathbf{r})\neq k_0$ [12]. In general Eqs. (1) and (2) have a unique solution precisely when the corresponding homogeneous pair of equations [$\rho_u(\mathbf{r})=0$ in Eq. (1)] has only the trivial solution $u(\mathbf{r})=0$. In such a case we say that the scatterer $\kappa(\mathbf{r})$ does not

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support internal resonances. In order to ensure unique solutions in what follows, we will take all scatterers considered henceforth to be of this kind.

Let us then assume that $f(\mathbf{r})$ is an arbitrary twice continuously differentiable function. Together with the field $u(\mathbf{r})$ we can use this function to define a function $u'(\mathbf{r})$ by

$$u'(\mathbf{r}) = \Theta(\mathbf{r})f(\mathbf{r}) + [1-\Theta(\mathbf{r})]u(\mathbf{r}), \tag{3}$$

where $\Theta(\mathbf{r})$ is an arbitrary twice continuously differentiable real function such that $\Theta(\mathbf{r}) \in [0,1]$, with $\Theta(\mathbf{r})=0$ and $\Theta(\mathbf{r})=1$ when $\mathbf{r} \notin \Omega$ and $\mathbf{r} \in \Omega'$, respectively. Here Ω' is a subregion of Ω with the property that $\partial\Omega' \cap \partial\Omega = \emptyset$, where $\partial\Omega$ and $\partial\Omega'$ are the boundaries of the regions Ω and Ω'. Since $f(\mathbf{r})$ and $\Theta(\mathbf{r})$ are twice continuously differentiable and since it can be shown that the piecewise continuity of the functions $\kappa(\mathbf{r})$ and $\rho_u(\mathbf{r})$ in Eq. (1) implies that the field $u(\mathbf{r})$ is twice differentiable [13], we can use Eq. (3) together with Eq. (1) to compute

$$\begin{aligned}
&\nabla^2 u'(\mathbf{r}) + \kappa^2(\mathbf{r})u'(\mathbf{r}) \\
&\quad = \Theta(\mathbf{r})[\nabla^2 + \kappa^2(\mathbf{r})]f(\mathbf{r}) - 4\pi[1-\Theta(\mathbf{r})]\rho_u(\mathbf{r}) \\
&\qquad + [\nabla^2\Theta(\mathbf{r}) + 2\boldsymbol{\nabla}\Theta(\mathbf{r})\cdot\boldsymbol{\nabla}][f(\mathbf{r}) - u(\mathbf{r})] \\
&\quad = -4\pi\rho_{u'}(\mathbf{r}),
\end{aligned} \tag{4}$$

where the last step defines the function $\rho_{u'}(\mathbf{r})$, which, in view of the properties of the constituent functions, is piecewise continuous. In addition, the properties of the functions $\rho_u(\mathbf{r})$ and $\Theta(\mathbf{r})$ imply that $\rho_{u'}(\mathbf{r})=0$ when $\mathbf{r} \notin \Omega$; i.e., the function $\rho_{u'}(\mathbf{r})$ is localized within the region Ω. Finally, since the scatterer $\kappa(\mathbf{r})$ is assumed not to support internal resonances, it follows that given $\rho_{u'}(\mathbf{r})$, Eq. (4) has a unique solution $u'(\mathbf{r})$, which by Eq. (3) is equal to $f(\mathbf{r})$ when $\mathbf{r} \in \Omega'$ and $u(\mathbf{r})$ when $\mathbf{r} \notin \Omega$. Together with Eq. (2) the latter of these properties implies that the field $u'(\mathbf{r})$ satisfies the Sommerfeld radiation condition.

To reiterate, we have constructed a source distribution $\rho_{u'}(\mathbf{r})$, which in the presence of the scatterer $\kappa(\mathbf{r})$ gives rise to a scalar field $u'(\mathbf{r})$. Within the region Ω' this field coincides with the function $f(\mathbf{r})$, whereas outside the region Ω it coincides with the field $u(\mathbf{r})$ generated by the source distribution $\rho_u(\mathbf{r})$. In the transition region $\Omega \setminus \Omega'$ the field is given by Eq. (3). We note that this kind of a construction is well known in the theory of nonradiating sources (see, e.g., Ref. [14]).

The second-order spatial coherence (correlation) properties of an ensemble $\{g\}$ of fields (functions) $g(\mathbf{r})$ are fully contained in the cross-spectral density (covariance) function [15,16]

$$W_g(\mathbf{r}_1,\mathbf{r}_2) = \langle g^*(\mathbf{r}_1)g(\mathbf{r}_2)\rangle, \tag{5}$$

where the angular brackets and the asterisk denote statistical averaging and complex conjugation, respectively. Suppose now that each source distribution in $\{\rho_{u'}\}$ is related to a field from the ensemble $\{u'\}$ through Eq. (4). Then we have

$$\begin{aligned}
16\pi^2 W_{\rho_{u'}}(\mathbf{r}_1,\mathbf{r}_2) &= \langle(-4\pi)\rho_{u'}^*(\mathbf{r}_1)(-4\pi)\rho_{u'}(\mathbf{r}_2)\rangle \\
&= [\nabla_1^2 + \kappa^{*2}(\mathbf{r}_1)][\nabla_2^2 + \kappa^2(\mathbf{r}_2)] \\
&\quad \times W_{u'}(\mathbf{r}_1,\mathbf{r}_2),
\end{aligned} \tag{6}$$

where ∇_i, $i=\{1,2\}$, operates on the position vector $\mathbf{r}_i$. The exchange of the order of differentiation and averaging is motivated by the existence of the ensemble average [16]. If we now assume that the fields in $\{u'\}$ are related to the fields in $\{u\}$ and the functions in $\{f\}$ by Eq. (3), and that these fields and functions are uncorrelated—i.e., $\langle f^*(\mathbf{r}_1)u(\mathbf{r}_2)\rangle=0$ when $\mathbf{r}_1 \in \Omega$, $\mathbf{r}_2 \notin \Omega'$—we get from Eq. (5) the expression

$$\begin{aligned}
W_{u'}(\mathbf{r}_1,\mathbf{r}_2) &= \langle u'^*(\mathbf{r}_1)u'(\mathbf{r}_2)\rangle \\
&= \Theta(\mathbf{r}_1)\Theta(\mathbf{r}_2)W_f(\mathbf{r}_1,\mathbf{r}_2) \\
&\quad + [1-\Theta(\mathbf{r}_1)][1-\Theta(\mathbf{r}_2)]W_u(\mathbf{r}_1,\mathbf{r}_2).
\end{aligned} \tag{7}$$

We then consider the degree of (spectral) coherence, which is defined for an ensemble $\{g\}$ by [15]

$$\mu_g(\mathbf{r}_1,\mathbf{r}_2) = \frac{W_g(\mathbf{r}_1,\mathbf{r}_2)}{\sqrt{W_g(\mathbf{r}_1,\mathbf{r}_1)W_g(\mathbf{r}_2,\mathbf{r}_2)}}, \tag{8}$$

with the convention $0/0=0$. The modulus of the degree of coherence is in the interval $[0,1]$, where the upper and lower limits correspond to complete coherence and complete incoherence, respectively [15]. When $\mathbf{r}_1 \in \Omega'$, we get from Eq. (7), for the degree of coherence of the ensemble $\{u'\}$, the expression

$$\mu_{u'}(\mathbf{r}_1,\mathbf{r}_2) = \begin{cases} \mu_f(\mathbf{r}_1,\mathbf{r}_2), & \mathbf{r}_2 \in \Omega', \\ \psi(\mathbf{r}_2)\mu_f(\mathbf{r}_1,\mathbf{r}_2), & \mathbf{r}_2 \in \Omega \setminus \Omega', \\ 0, & \mathbf{r}_2 \notin \Omega, \end{cases} \tag{9}$$

where $\psi(\mathbf{r}) \in [0,1]$. Hence, for points inside the region Ω', the degree of coherence of the fields $\{u'\}$ is equal to that of the functions $\{f\}$. In addition, its modulus for a point in Ω' and a point outside that region never exceeds the modulus of the corresponding degree of coherence of $\{f\}$. Finally, when $\mathbf{r}_1, \mathbf{r}_2 \notin \Omega$, Eqs. (7) and (8) imply that $\mu_{u'}(\mathbf{r}_1,\mathbf{r}_2) = \mu_u(\mathbf{r}_1,\mathbf{r}_2)$; that is, for points outside the region Ω the degree of coherence of $\{u'\}$ equals that of $\{u\}$. Thereby the coherence properties of the ensemble of fields $\{u'\}$, corresponding to the constructed ensemble of source distributions $\{\rho_{u'}\}$, are completely determined inside Ω' by the coherence properties of the ensemble of arbitrary twice continuously differentiable functions $\{f\}$.

The coherence length of an ensemble of functions (fields) $\{f\}$ is roughly the distance over which correlations between pairs of points are significant—i.e., where the degree of coherence $\mu_f(\mathbf{r}_1,\mathbf{r}_2)$ differs appreciably from zero [15]. This definition is necessarily vague by nature, since not all functional forms that the degree of coherence may have can be assigned a coherence length in a meaningful way. The situation is reminiscent of the difficulty to define the effective spectral range of multimode radiation [15]. However, in the specific case when the degree of coherence is of sinc type, or

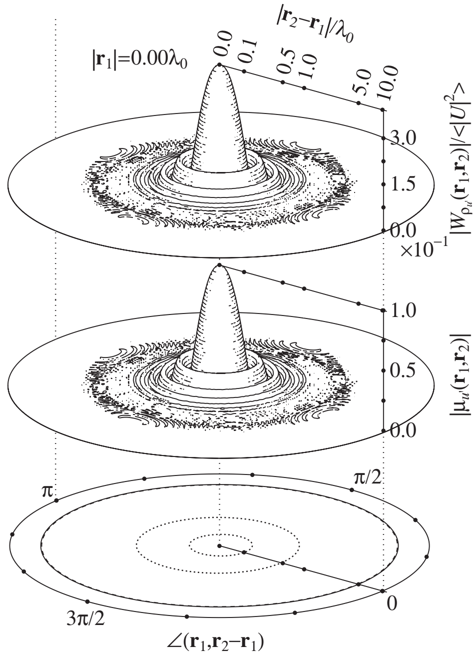

FIG. 1. The source cross-spectral density $W_{\rho_{u'}}(\mathbf{r}_1,\mathbf{r}_2)$ (top) and the degree of coherence $\mu_{u'}(\mathbf{r}_1,\mathbf{r}_2)$ of the generated field (middle) around $\mathbf{r}_1$, when $|\mathbf{r}_1|=0.00\lambda_0$.

$$\mu_f(\mathbf{r}_1,\mathbf{r}_2)=j_0(\chi|\mathbf{r}_1-\mathbf{r}_2|)=\mathrm{sinc}(\chi|\mathbf{r}_1-\mathbf{r}_2|), \qquad (10)$$

where $\chi\in\mathbb{R}$ and $j_0(z)$ is the spherical Bessel function of zeroth order, the coherence length is conventionally taken to be precisely $L=\pi/\chi$ [3,15]. Therefore, to avoid getting tangled up in a vague definition, we concentrate here only on ensembles with a degree of coherence that is of the form of Eq. (10) inside the region Ω'. Hence they are in particular statistically homogeneous and isotropic therein.

Let us now consider a situation where the regions Ω and Ω' are balls of radii R and $R'<R$, respectively, and $\kappa(\mathbf{r})=\kappa$ when $\mathbf{r}\in\Omega$. Furthermore, we assume that $W_f(\mathbf{r},\mathbf{r})=1$ for all $\mathbf{r}$, so that the definition (8) implies that $W_f(\mathbf{r}_1,\mathbf{r}_2)=\mu_f(\mathbf{r}_1,\mathbf{r}_2)$. In addition, we take the fields in the ensemble $\{u\}$ to be given by

$$u(\mathbf{r})=\begin{cases} U\{[j_0(\kappa r)-j_0(\kappa R)]h_0^{(1)\prime}(k_0R) \\ \quad +\kappa/k_0 j_0'(\kappa R)h_0^{(1)}(k_0R)\}, & r<R, \\ U\kappa/k_0 j_0'(\kappa R)h_0^{(1)}(k_0r), & r>R, \end{cases} \qquad (11)$$

where U, with $\langle f^*(\mathbf{r})U\rangle=0$ for all $\mathbf{r}$, is a random variable that alone indexes the realizations, $h_0^{(1)}(z)$ is the spherical Hankel function of zeroth order, and the primes denote differentiation. The fields $u(\mathbf{r})$ thus defined correspond to constant source distributions $\rho_u(\mathbf{r})=\rho_u$ within Ω. Finally, if we also assume that in the transition region $R'<r<R$ we have $\Theta(\mathbf{r})=p\left(\frac{r-R'}{R-R'}\right)$, where $p(t)=-6t^5+15t^4-10t^3+1$, we can use Eqs. (11), (5), (7), (6), and (8) to determine the functions $W_{\rho_{u'}}(\mathbf{r}_1,\mathbf{r}_2)$ and $\mu_{u'}(\mathbf{r}_1,\mathbf{r}_2)$. The magnitudes of these functions are plotted in Figs. 1–3 for the specific case when $\kappa/k_0=1.1$ ($\lambda\approx0.91\lambda_0$), $R=5.0\lambda_0$ ($\approx5.50\lambda$), $R'=4.9\lambda_0$ ($\approx5.39\lambda$), $\chi/k_0=5$ ($\chi/\kappa\approx4.55$), and $\langle|U|^2\rangle=100\pi^2$. We observe that this choice of parameters implies that the scattering ball $\kappa(\mathbf{r})$ does not support internal resonances and hence the constructed source cross-spectral density $W_{\rho_{u'}}(\mathbf{r}_1,\mathbf{r}_2)$ indeed uniquely gives rise to the degree of coherence $\mu_{u'}(\mathbf{r}_1,\mathbf{r}_2)$ of the generated fields $\{u'\}$. In the plots these quantities are shown as functions of the logarithmically displayed distance $|\mathbf{r}_2-\mathbf{r}_1|/\lambda_0$ and the angle $\angle(\mathbf{r}_1,\mathbf{r}_2-\mathbf{r}_1)$ for the cases when $|\mathbf{r}_1|/\lambda_0\in\{0.00,2.50,4.89\}$. The solid and dashed curves in the bottom part of each figure correspond to the outlines of the regions Ω and Ω', respectively. The dot-

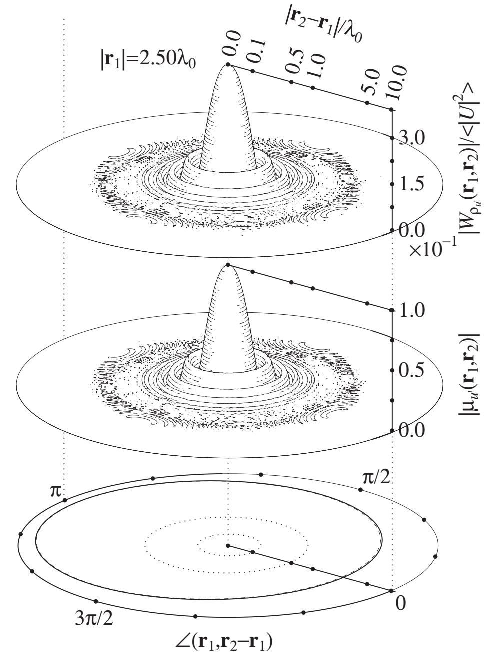

FIG. 2. Same as Fig. 1, but with $|\mathbf{r}_1|=2.50\lambda_0$.

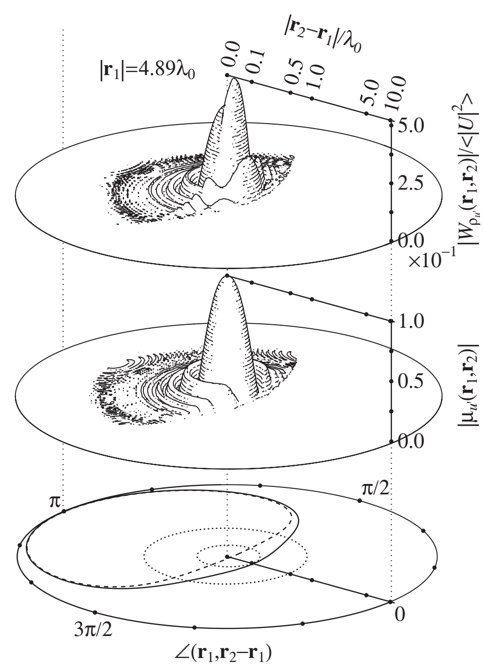

FIG. 3. Same as Fig. 1, but with $|\mathbf{r}_1|=4.89\lambda_0$.

ted curves in turn correspond to the first zeros of $\mu_{u'}(\mathbf{r}_1,\mathbf{r}_2)$ (inner) and $j_0(k_0|\mathbf{r}_1-\mathbf{r}_2|)$ (outer). These figures clearly show that inside the region Ω' the coherence length L of the fields $\{u'\}$ is much shorter than the coherence length $L_0=\pi/k_0=\lambda_0/2$ corresponding to the latter function. In fact, from Eq. (10) it follows that $L=\pi/\chi=\pi/(5k_0)=\lambda_0/10$.

In the example we have used as a yardstick the vacuum wavelength λ_0, rather than the wavelength λ associated with the field within the source region. The wavelength inside a source or a scatterer is, in general, not unambiguously defined since the refractive index may be position dependent or complex valued (lossy medium). This is, though, not the case in the analyses in Refs. [1–3,5,6], in which a lossless medium is taken to fill all space (including the source region). We emphasize that since the general construction put forward in this paper can be used to obtain field coherence properties (including the coherence length) that are independent of the vacuum wavelength λ_0 and the refractive-index distribution $\kappa(\mathbf{r})$, they are also independent of the wavelength in the medium. For lossless and homogeneous media (scatterers) $\kappa(\mathbf{r})=\kappa=nk_0$, so that $\lambda=\lambda_0/n$. Thereby, although we compare the coherence length in the example to the *vacuum* universal form $j_0(k_0|\mathbf{r}_1-\mathbf{r}_2|)$, the conclusions derived from the example are not altered if k_0 (λ_0) is replaced by κ (λ).

The significance of the presented example lies in the fact that it shows that even when the source region is large and the source distribution is statistically homogeneous and isotropic, the field correlations are not of the universal sinc-function form $j_0(k_0|\mathbf{r}_1-\mathbf{r}_2|)$ [or $j_0(\kappa|\mathbf{r}_1-\mathbf{r}_2|)$] in a lossless medium, contrary to what could be expected from the results in Refs. [2,5]. Indeed, the source distribution is statistically homogeneous and isotropic within the smaller ball Ω', and the radius of the source region in the example is $5\lambda_0$, which by Ref. [1] constitutes a large region (even if the radius is interpreted as 5.50λ) at least when $\mathbf{r}_1=\mathbf{0}$. Furthermore, an analogous construction can be used for balls with arbitrarily large radii. Although the correlations of the source distribution are within the region Ω' exactly of the form considered in Ref. [3], they are, however, more complex inside the boundary region $\Omega\backslash\Omega'$. It is precisely the existence of such a region that invalidates the universality results. That is, the assumption of an infinite source region, as used in obtaining the universality results, prohibits the existence of any boundaries and hence cannot account for any effects due to them. In view of these considerations and since any actual region, however large, has a boundary, it thereby seems that the applicability of the universality results to large finite systems should be carefully evaluated on a case-by-case basis.

It is also of interest to note that the coherence length of the field in the example is $\lambda_0/10$, which is much less than the sinc-function (universal) coherence length $\lambda_0/2$ (or $\lambda/2$), which also corresponds to a field produced by an infinite δ-correlated source [1]. In fact, since there is no restriction to the parameter χ in Eq. (10), it is possible to construct a source distribution which yields a field with an arbitrarily short coherence length inside a region with any specified refractive-index distribution, lossy or not. In particular, this coherence length does not depend on the refractive-index distribution or on the wavelength of the field (however it is defined) within the source region. This is in contrast to the short coherence length obtainable in a lossy system with a δ-correlated source distribution, where the coherence length can be made arbitrarily short by increasing the losses in the system since absorption reduces the effective range of the contribution to the field as produced by each source point [7].

The fact that the field can have an arbitrarily short coherence length in a finite lossless system, although obvious from the equations derived here, is perhaps not so from a physical point of view. Indeed, the idea that blackbody or δ-correlated sources produce the fields with the shortest coherence lengths is entertained implicitly and explicitly in, for example, the seminal papers, Refs. [1,3]. In the former of these references it is argued that for the coherence length of the field to be shorter than $\lambda/2$, the correlations of the source distribution must be more singular than the δ function. The results and the example in the present paper show this conclusion to be false, since the short coherence length is obtainable even for a piecewise continuous source distribution. In Ref. [3], on the other hand, an explicit example of a shorter coherence length is obtained, but it is dismissed as an extreme case of the $\lambda/2$ bound for fields whose cross-spectral density function is not of the sinc form. There is also some anecdotal evidence that the $\lambda/2$ limit has been even more widely adopted. One reason for this is probably the universality results [2,5], which predict a $\lambda/2$ coherence length for fields produced by a wide class of source distributions. Another likely reason is that the $\lambda/2$ limit also appears in considerations of so-called free fields. The Fourier transform of such fields over all space is concentrated on a spherical shell of radius $k_0=2\pi/\lambda_0$, and it is reasonable to assume that no collection of such fields could harbor structures that are significantly smaller than the wavelength λ_0. This fact is proven for an isotropic ensemble of uncorrelated plane waves in Ref. [4].

Even though the analysis in this paper is based on scalar fields, the generalization concerning the universality results of infinite statistically homogeneous and isotropic source regions to a full vectorial description of electromagnetic fields [6] suggests that it should be straightforward to extend our results to hold for vectorial fields as well. Indeed, the explicit construction of a source distribution that yields a field with prescribed properties, on which our conclusions are based, is not only available for scalar fields but can also be performed in the realm of a complete vector-valued description of electromagnetic fields [17].

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[12] This fact can be proven by considering the difference $\Delta u(\mathbf{r})$ of two solutions to Eqs. (1) and (2), integrating the function $|\partial_r \Delta u(\mathbf{r}) - ik_0 \Delta u(\mathbf{r})|^2$ over an infinite spherical surface and subsequently applying Eq. (2), Green's theorem, and Eq. (1) to obtain $(\nabla^2 + k_0^2)\Delta u(\mathbf{r}) = 0$ for all $\mathbf{r}$, whereby Eq. (2) implies that $\Delta u(\mathbf{r})$ must vanish identically [11].
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